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Principles for Optimizing Geometric and Functional Properties of Railway Track Reverse Transitions

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Abstract: The article describes the principles of optimizing the railway track alignment design on the example of a transition between reverse curves. The purpose of this optimization was to minimize the length of the reverse transitions of railway track under the condition of ensuring the simplicity of their construction and subsequent operation with a high level of safety and driving comfort. The results obtained show that this goal can be achieved by using consistent forms of inverse transitions with non-identical functions of track axis curvature variation and its cross slope. This is confirmed by comparing the achieved level of their quality with the quality of similar transitions designed using "out-swinging" transition curves. The potential of their advantages is limited by some provisions of the current regulations with formal requirements for the design parameters of the alignment of the railway track. In contrast to this, they should contain requirements for integrative indicators of the ergonomic and technical aspects of the quality of the Track + Rail Car system functioning. They should be calculated taking into account the kinematics of the rail car design points located at levels that are relevant for these aspects. For example, at the level of the highest location of the vestibular apparatus of the passenger and/or the center of mass of the rail car. It is also important to take into account the kinematics of the points of interaction of the coupling devices of the rail cars. The examples of optimization of reverse transitions considered in the article indicate a significant influence of the geometric properties of their shapes on the coordinates, curvature and accelerations at these points. From this it follows that in order to implement the progressive requirements for taking into account their kinematics, the following are necessary: a) criteria for the compliance of its indicators with the conditions of safe and comfortable movement; b) requirements for properties of the curvature and cross slope functions that may be suitable for transitions design; c) criteria for harmonizing these properties in order to achieve the best result under given constraints. The issues considered in this article can serve as a basis for a discussion about the feasibility and effectiveness of implementing these measures for consistent forms of reverse transitions with non-identical variant functions of the track curvature and its cross slope.

Keywords: System Track+Rail Car, Alignment of Transition, Harmonization of System Properties, Kinematics of Body Design Point

1. Introduction

Optimization of geometrical and functional properties of the railway transitions is a very topical issue. This is indicated by a large number of studies [1-4] and ever-growing interest in this problem belonging to promising research areas [5]. The relevance of solving this and other problems of geometric design of rail track increases significantly due to the objective need to increase the speed, safety and comfort of traffic on the existing and intensively developing network of new speed and high-speed railways.

To solve these tasks, it is necessary to improve a number of requirements of the current standards which are based on the experience of operating only clothoid forms transitions, in particular, requirements of the cant gradient D/L and rate of change of cant v×D/L. It is clear that the values of D/L and v×D/L can sometimes correlate with the quality of curvilinear motion of rail car with variable curvature and body roll. However, the causes and consequences of this correlation are not theoretically substantiated either for clothoidal or for half-sine shape transitions. Influence of the dominate significance of the parameter D/L and/or v×D/L does not allow one to take into account a very significant factor of the interaction of the nonlinear differential properties of their functions f_k and f_ψ and its impact on the quality of the curvilinear motion of rail cars. This leads to excessive elongation of half-sine shape transitions and makes them difficult to use in practice.

In the research results published previously [6-8] recommendations for progressive design of track alignment and assessment of its quality considering the kinematics of the center of mass of the rail car [5] have been taken into account. Analysis of these results shows that the lengths of the new harmonized forms of transitions differ lengths of significantly from those of the shapes of transitions with constant or not constant gradient of curvature and cross slope, whose has been set in these standards. This necessitates further research of the possibility and features of using harmonized forms of different types of transitions in progressive design for optimum track alignment when constructing new or reconstructing existing curves. The principles of their optimization between inverse curves with $R_B \times R_E < 0$ are discussed in this article.

2. Purpose and Principles of Progressive Track Alignment Design

The purpose of the progress declared in [5], which is recommended to be achieved by using only "out-swinging" transition curves, is not explicitly stated. Also, this recommendation significantly limits the list of options that can be used to perform progressive path alignment, taking into account the level of the center of mass and/or other functionally significant levels of the rail car. Obviously, the progressive track alignment should ensure the simplicity of its construction and subsequent operation with a high level of safety and driving comfort at a given speed V with the minimum necessary, but sufficient length for this. Allowing for the solutions obtained earlier [6, 7] and the recommendations given in [5] this problem can be decomposed into a number of parts solved by the observance of the principles that are described below.

2.1. The Principle of Continuity of Geometric Properties of Reverse Transition

This principle is observed by conjugating the reverse circles track sections with curvature K_B, K_E and cross slopes Ψ_B, Ψ_E by a single transitional section with a symmetrical nonlinear derivation of elevations D_B and D_E, which correspond to the condition of proportionality $K_B/K_E \approx D_B/D_E$. This principle also assumes that the geometrical form of the reverse transition must be described by a pair of non-identical normalized mathematical functions $f_\psi(p)$ and $f_k(p)$. At the same time, the current values of the cross slope of the track $\psi(l)$, the horizontal curvature of its axis $k(l)$, as well as the deviations of the values of elevations along the left $d_L(l)$ and right $d_R(l)$ rails from the design marks of the longitudinal profile of the track axis along the entire length L must be calculated by the following formulas:

$$\psi(l) = \Psi_B + f_\psi(p)\Delta\Psi \quad (1)$$

$$k(l) = K_B + f_k(p)\Delta K \quad (2)$$

$$d_L(l) = \psi(l)B/2 \quad (3)$$

$$d_R(l) = -d_L(l) \quad (4)$$

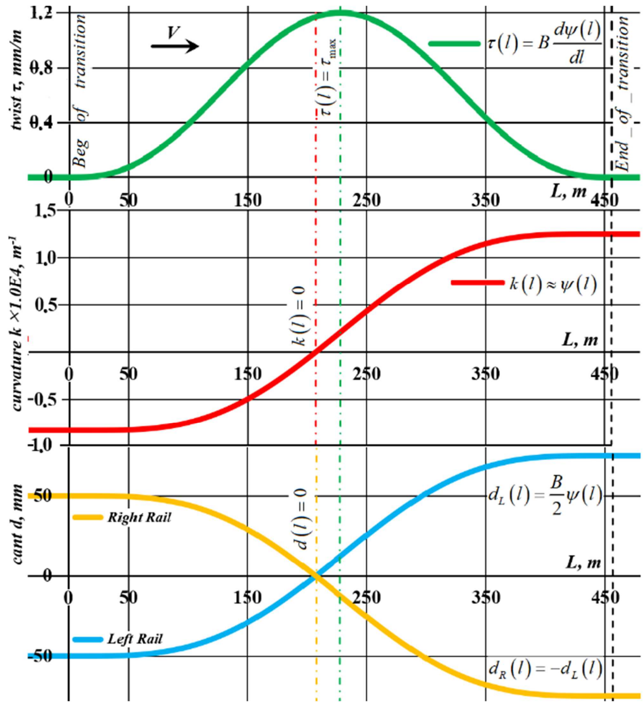

Figure 1. *Continuity of non-identical non-linear gradients of curvature and cross-slope of the track, as well as the symmetrical transition of the elevations of its rails at $K_B \times K_E < 0$, and $|K_B| < |K_E|$ and $K_B \times D_B = K_E \times D_E$.*

2.2. The Principle of Harmonizing of Geometric Properties of Reverse Transition

The provisions of the continuity principle create prerequisites for the observance of additional conditions for minimizing the length of the reverse transition by harmonizing the interaction between the geometrical properties of the non-identical normalized functions f_ψ (p) and f_k (p). In common terms, these conditions can be formulated as the ones that ensure maximum comfort of passenger movement and the smoothest change in the forces of interaction between the elements of the system Track + Train. The difficulty of predicting and assessing the degree of compliance with these conditions is due both to the heterogeneity of the elements of the system Track + Train, and the dependence of the results of their interaction on the simultaneous or sequential action of a large list of deterministic and probabilistic factors. Therefore, compliance with the above conditions should be assessed by the degree of elimination or minimization of the negative impact of those deterministic factors that are caused by the geometric properties of the reverse transition for given values of the design vehicle parameters and its speed.

This makes it possible to significantly simplify the model of the Track + Train system due to the separate consideration of the influence of these factors on rail cars as on part of their chain and their impact on a rail car as on an individual element of the subsystem Track + Rail Car. The condition for such a simplification is the interaction of adjacent cars of the train only at the point of contact of their coupling devices. Usually, these points are substantially removed from the central fixing pins of the rail cars. The relative value of this offset may be to 1/5 of the distance between the central pins of the locks of one rail car. In this case, the weight of the negative result of the interaction of lateral forces between coupling devices of adjacent rail cars depends on the difference of coordinates and curvature of trajectories of their contact points calculated at conditionally 'free movement' of each of them (Figure 2).

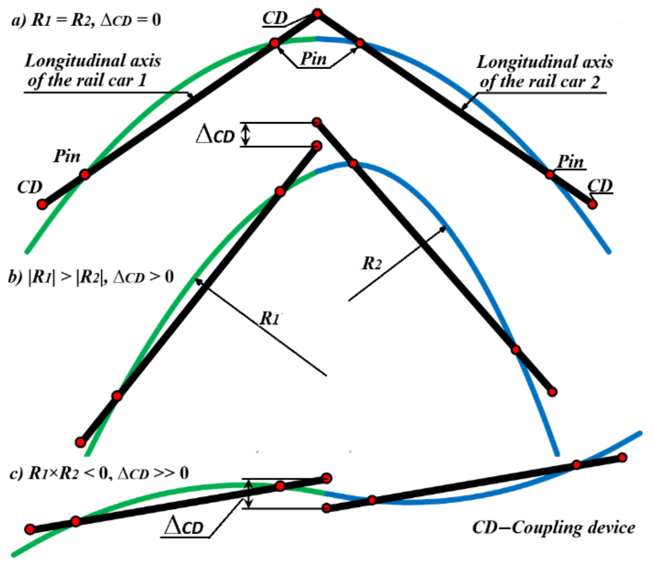

Figure 2. *Kinematics of the contact points of couplings of rail cars at different orders of conjugation smoothness and curvature of adjacent elements of the track axis.*

According recommended to the equation [5], the curvature of the movement trajectory of each kingpin at the level of the rail car hitch $H_{CD} \approx 1100$ mm can be described by a simplified formula

$$k_H(l) \approx K_B + f_k(p)\Delta K + \left(d^2 f_\psi / dl^2\right) H_{CD} \Delta\Psi \quad (5)$$

From the kinematics of the contact points and this formula, it follows that the difference in coordinates and the curvature of the trajectories of the "free" movement of the coupling devices points of each of the neighboring cars can be significantly affected by:

1) order of continuity G^n of conjugations of adjacent elements;
2) their curvature k(l);
3) curvature gradient dk(l)/dl;
4) difference of track cross slopes of the transition $\Delta\Psi$;
5) elevation H_{CD} of the coupling device above the top of the heads of the rails of the track;
6) differential properties of the function of cross slope reverse transition track d^2f_ψ/dl^2.

An unfavorable combination of their properties can cause undesirable episodes of "hunting" some rail car designs, discomfort in the movement of passengers and even the "displacement" from the track of relatively lighter or empty rail cars of train. This also follows that the risks of negative impact of the interaction of elements of the Track + Train system on the quality of its operation can be significantly reduced by providing continuous and smooth trajectory of the coupling points at the H_{CD} level. Accounting for this feature of the Track + Train system at harmonizing the geometric properties of the reverse transition enables one to simplify the achievement of its goals using a simpler model of the Track + Rail Car system. At the same time, the influence of the geometric properties of the reverse transition on the integrative performance indicators of the Track + Rail Car system should be evaluated taking into account the curvature of the trajectories of points at all its significant levels H_{AB}, H_{CD}, H_{MC} and H_{VAP} without considering the springs [5].

These conditions are taken into account in the harmonization method [7]. This method allows one to calculate such lengths L of the reverse transition, in which the interaction of the geometric properties of its shape minimizes the factors of a negative impact on safety and driving comfort due to:

1) observing the necessary and sufficient geometric smoothness of the conjugation of the functions $f_\psi(p)$ and $f_k(p)$ with the functions of similar properties of adjacent track elements;
2) providing the most appropriate forms of diagrams of changes in the integrative indicators of the quality of the functioning of the Track + Rail Car system at the H levels relevant for each of its aspects;
3) preserving these properties under changing the length L to comply with the permissible values of passenger comfort limits $J_{max} \le J_{lim}$ and track twisting $\tau_{max} \le \tau_{lim}$.

This result of the interaction between the non-identical properties of the functions $f_\psi(p)$ and $f_k(p)$ is largely predetermined by specially chosen coefficients of their non-standard polynomials G^3 and G^4 orders of continuous. These orders of continuous of the functions $f_\psi(p)$ and $f_k(p)$ remain unchanged when correcting their values depending on the values of the parameters Z and U and the additional functions $Z\varepsilon_\psi(p)$ and $U\varepsilon_k(p)$. The need for such an adjustment increases as the current design value parameters V, R, and D deviate from those that provided the values of non-compensated lateral acceleration in the running plane at $|a| \ge 0.3$ m/s^2. The values of parameters U and Z are set during the harmonization process together with the search for the necessary and sufficient length L of the reverse transition for given values of V, ΔK, $\Delta\Psi$, B and H_{Des}.

The algorithm for solving this problem is described in [9]. In the process of its solution, the current and final values of the functions of the harmonized reverse transition form are determined as the result of adding their basic values $f_\psi(p)$ and $f_k(p)$ with corrections to them $U\times\varepsilon_k(p)$ and $Z\times\varepsilon_\psi(p)$ (Figure 3).

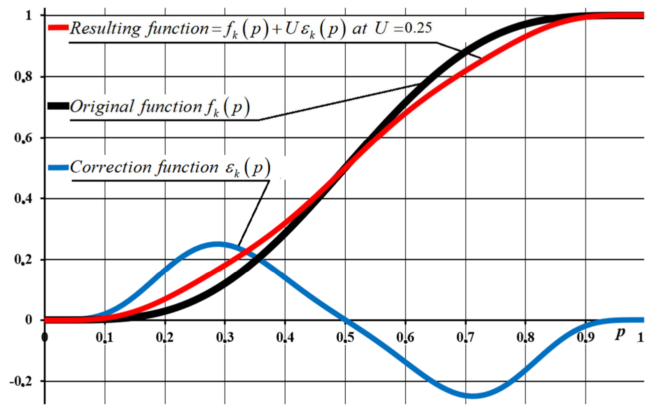

***Figure 3.** An example of correcting the values of the base normalized function $f_k(p)$ in accordance with the values of the corrective function $U\varepsilon_k(p)$.*

2.3. The Principle of Compliance with the Requirements of Ergonomic and Technical Aspects of Quality of the Track+Rail Car System

Harmonization of the values of the parameters L, U and Z creates the prerequisites for meeting the requirements of the ergonomic and technical aspects of the quality of the functioning of the Track + Rail Car system for the given values of the parameters V, ΔK, ΔΨ, B and H_{Des}. First of all, these prerequisites are ensured by a continuous, smooth and strictly monotonic change in the non-compensated lateral acceleration $a_H(l)$ acting throughout the entire reverse transition at the calculated rail car level $H = H_{Des}$. This is most beneficial for minimizing the maximum rate of its change J_{max} => MIN.

The ratio of the achieved value J_{max} and its limit J_{lim} makes it possible to judge the compliance with the requirements for driving comfort according to the condition $J_{max} \le J_{lim}$. Therefore, the harmonization of the properties of normal, oval or reverse transitions should be started at $H_{Des} = H_{VAP}$. In this case, the plot of the function j = da/dt acting at the H_{VAP} level will acquire a trapezoidal shape that is ideal for comfort. And at all levels $H < H_{Des}$, the diagrams of the functions $j_H(l)$ will have bell-shaped forms that also are very favorable for the technical aspects of quality.

This feature of harmonized forms of transitions is rationally consistent with the ratio of functionally significant levels of working rail cars $0 < H_{AB} < H_{CD} < H_{MC} < H_{VAP}$. The solutions obtained with $H_{Des} = H_{VAP}$ must be checked for compliance with the comfort criterion $J_{max} \le J_{lim}$ and the allowable track twisting $\tau_{max} \le \tau_{lim}$. The value of J_{max} required for this check is calculated in the middle of the harmonized reverse transition form at l=L/2 and p = 0.5 by the formula.

$$j_{max} = \Big(\big(\Delta K(df_k/dl + U\,d\varepsilon_k/dl) + H\Delta\Psi(d^3f_\psi/dl^3 + Z\,d^3\varepsilon_\psi/dl^3)\big)v^2 - g\Delta\Psi(df_\psi/dl + Z\,d\varepsilon_\psi/dl)\Big)v \quad (6)$$

The τ_{max} value traditionary is calculated at p = 0.5, taking into account the length of the base b, which should vary from 3.0 m to 20 m. These calculations performed using the following formula.

$$\tau_{max} = B|\Delta\Psi\Big(\big(f_\psi(p + b/(2L)) + Z\varepsilon_\psi(p + b/(2L))\big)\Big) - \big(f_\psi(p - b/(2L)) + Z\varepsilon_\psi(p - b/(2L))\big)|/b \quad (7)$$

At a low design speed V, the value τ_{max} may exceed the limit value τ_{lim} set for him. Such a disadvantage is eliminated with at rational combination of the values of the parameters R_B, D_B, R_E, D_E circular parts of the reverse transition with the calculated movement speed V and with the nominal value H_{Des}. The dependences of the values of the criteria J_{max} and τ_{max} on these parameters are described in article [8]. When the $H_{Des} > H_{VAP}$ level increases, the J_{lim} and/or τ_{lim} limits can only be met by lengthening the harmonized reverse transition. And with an increase in the radii R_B, R_E and a decrease of cants D_B, D_E, they can be met by a decrease in its length L. The consequences of these measures are illustrated in Examples 2 (Figure 5) and 3 (Figure 6). Depending on the track alignment design conditions, their combination can also be effective.

3. Examples of Optimizing Harmonized forms of Reverse Transitions

The above principles are universal and applicable for the construction of normal (with K_B=0 or K_E=0), oval (with $K_B \times K_E > 0$) or reverse transitions (with $K_B \times K_E < 0$). The functions $f_k(p)$, $f_\psi(p)$, $\varepsilon_k(p)$, $\varepsilon_\psi(p)$, methods for their coordination and calculation of rectangular coordinates x(l), y(l) of the axis, which are necessary for constructing and for maintaining of the geometry of railway transitions are also universal. The features of the application of these principles have been studied in detail for a large number of options of the initial data for normal type of the transition of 1520 mm track [8]. Selective harmonization of normal transitions with the same parameters in the rounding of the 1435 mm track led to almost the same results. This made it possible to significantly reduce a number of similar studies on the harmonized reverse transitions for the 1435 mm track.

The track alignment using traditional geometry in the form of two clothoids directed in opposite directions have the same functional disadvantages that are inherent in normal transitions with constant gradient of curvature and cant. Therefore, even if the parameters A of these clothoids are equal, the length of such reverse transitions is increased to create straight sections between them. This is necessary to dampen the oscillations of the rail cars of the train after they pass through the normal transitions of an imperfect clothoid shape. In some high-speed railway projects, the length of such sections reverse of transitions reaches 1.5 km or more. The results of the shape harmonization of the railway track reverse transitions show that, subject to the principles outlined in this article, the lengths of such sections can be several times shorter than traditional ones. Therefore, their use can significantly expand the possibilities of tracing of more economical, safe and convenient railways. This is especially true in conditions of rugged and built-up area saturated with other communications.

With this in mind, one of the objectives of this study was to evaluate the differences in the properties of the harmonized reverse transition from those of analogues with only modern geometry. To evaluate these differences, the calculations were performed with the following predefined parameter values: B = 1500 mm, H_{VAP} = 2200 mm, H_{MC} = 1800 mm, H_{CD} = 1100 mm, H_{AB} = 460 mm и J_{lim} = 0.4 m/s^3. The limit of allowable torsion of the railway track τ_{lim} was set at 1 mm/m below than the minimum «AL» level perceptible in use. Taking into account the requirements [10], the τ_{lim} values were taken to be 3 mm/m and 2 mm/m for speeds V ≤ 200 km/h and V > 200 km/h, respectively. The calculation results with these data are presented by the diagrams and numerical values of the properties corresponding to low (V = 120 km/h) and high (V = 400 km/h) movement speeds.

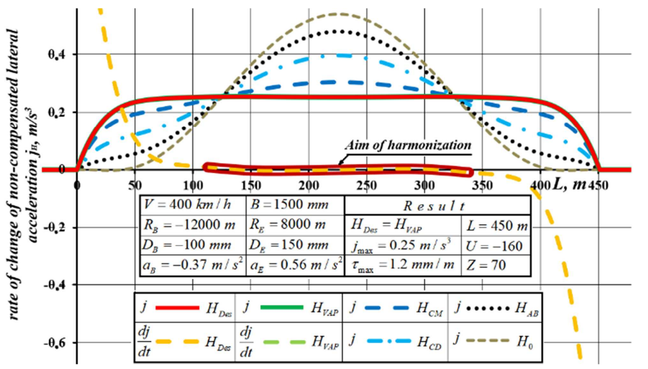

Figure 4. The Results of the shape harmonization of reverse transition (Example N 1).

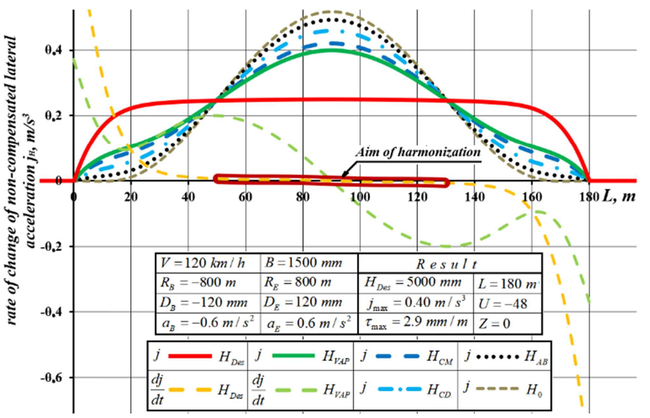

Figure 5. Results of the shape harmonization of elongated (from 120 m to 180 m) reverse transition at Hdes = 5000 mm to comply with the condition τmax ≤ 3 mm/m (Example N 2).

The differences of the harmonized properties of the reverse transition important for comparison are shown by the diagrams of the functions j (l) acting at the functionally significant levels H_{Des}, H_{VAP}, H_{MC}, H_{CD}, H_{AB} and H = 0. They are supplemented by plots of the function $dj_H(l)/dt$ calculated at the H_{Des} and H_{VAP} levels. In the figures with diagrams of the functions $j_H(l)$, they are shown by a dotted line. Their values on the ordinate axis are given in m/s^4. The part of the diagrams of functions $dj_H(l)/dt$ of the H_{Des} level, the values of which are taken into account in the formalized function of the purpose of harmonization of reverse transition [7], is shown with a solid line. The less this portion of the function $dj_H(l)/dt$ deviates from the abscissa axis, the more similar to a trapezoid the shape of the corresponding diagram of the functions $j_H(l)$ will be. This indicates that the non-compensated lateral acceleration $a_H(l)$ acting at the design level H_{Des} will also change more smoothly and strictly monotonically. At H_{Des} = H_{VAP}, the same ideal movement conditions will be inherent in the H_{VAP} level.

In this article the figures with these diagrams are supplemented by the values of corresponding individual initial data V, R_B, D_B, R_E, D_E, H_{Des} and the calculated parameters L, U, Z, as well by the criteria for assessing passenger comfort J_{max} and torsion of the track τ_{max}.

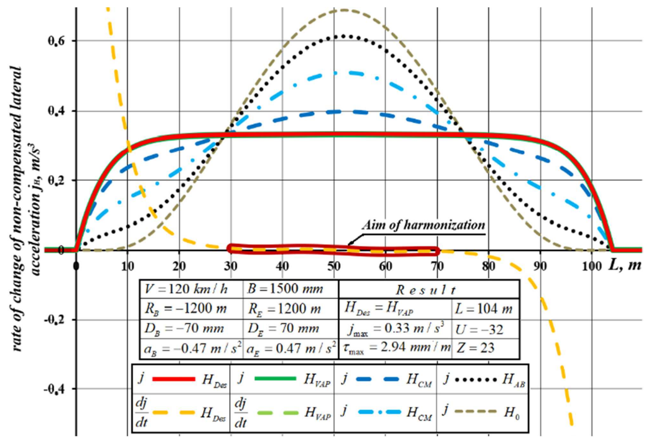

Figure 6. Results of compliance with the condition τmax ≤ 3 mm/m when harmonizing the shape of a shortened (from 120 m to 104 m) reverse transition, achieved at increased (relative to the data of Example No. 2) radii RB, RE and at reduced values DB, DE of the cants at the nominal level H_{Des} = H_{VAP} (Example N 3).

This detail level of the harmonization results of the examples given in this article is due to its cognitive purposes. For practical purposes, this number of diagrams is redundant. It can be reduced to two diagrams $j_H(l)$ operating at the H_{Des} and H_{VAP} levels of the rail car. In addition, they can be supplemented with the corresponding values of the parameters L, U, Z, J_{max} and τ_{max}.

4. Comparison of Alternatives Modern Designs of the Reverse Transitions

As a result of modern research and the search for better transition designs, a new type of "out-swinging" transition curve appeared at the beginning of this century. Generally, this term means that the transition curve has such a design feature that prevents the body of rail car from swaying. To do this, the curvature of the axis of the rail track transition is defined as a kind of 'antipode' of the curvature of the trajectory of the mass center of the 'rigid' rail car. In contrast to this, in harmonized transitions, the kinematics of the rail car design point is optimized at G^4-th continuous, strictly monotonous, half-sinusoidal curvature of the track axis (Table 2).

The first successfully tested "out-swinging" transition curve was called "Viennese transition curve and cant gradient" type HHMP7 [11]. The abbreviation HHMP7 denotes one of the 4 functions f(p) [12]. This abbreviation determines the identical dependence of the values of the normalized multiplier for the cross slope $\Delta\Psi$ of the rail car and curvature ΔK of the trajectory of its mass center. It is a standard polynomial with a minimum 4-th degree of the first term and a maximum 7-th degree of the last, fourth term.

Table 1. *List of symbols and abbreviations.*

Symbol	Designation	Unit
A	smoothness parameter, the square whose value is inversely proportional to the curvature gradient of the clothoid	m
a_B	non-compensated lateral acceleration in the beginning of transition	m/s^2
a_E	non-compensated lateral acceleration in the end of transition	m/s^2
$a_H(l)$	current value of the non-compensated lateral acceleration at level H at a distance l	m/s^2
B	distance between wheel treads of an axle (on track with gauge 1435 mm: B = 1500 mm);	mm
D_B	cant in beginning of transition	mm
D_E	cant in end of transition	mm
ΔD	difference of the cants $\Delta D = D_E - D_B$	mm
$d_L(l)$	current elevation of the left rail head above the calculated profile of its track axis	
$d_R(l)$	current elevation of the right rail head above the calculated profile of its track axis	
dd/dl	cant gradient	mm/mm
dd/dt	rate of change of cant	mm/s
G^n	order of geometric continuity of conjugation of diagrams of two functions, determined by the largest number n of their derivatives with equal values at a common point for them	–
g	acceleration due to gravity	m/s^2
L	length of transition design of railway track (include curve and cant)	m
l	current value of the longitudinal distance from the beginning of the transition, $0 \leq l \leq L$	m
R_B	radius of horizontal curve of track axle in begin of transition	m
R_E	radius of horizontal curve of track axle in end of transition	m
p	proportion of the current value of the longitudinal distance p = l/L	–
t	time	s
V	sped	km/h
v	sped	m/s
K_B	horizontal curvature of track axle in begin of transition $K_B = 1/ R_B$	m^{-1}
K_E	horizontal curvature of track axle in end of transition $K_E = 1/ R_E$	m^{-1}
ΔK	difference of the curvature of reverse transition $\Delta K = K_E - K_B$	m^{-1}
k(l)	current value of the curvature of track axle at a distance l	m^{-1}
$k_H(l)$	current value of the curvature of the trajectory of the point at a level H at a distance l	m^{-1}
Ψ_B	track cross slope in begin of transition $\Psi_B = D_B/B$	–
Ψ_E	track cross slope in end of transition $\Psi_E = D_E/B$	–
$\Delta\Psi$	difference of track cross slopes of the transition $\Delta \Psi = \Psi_E - \Psi_B$	–
$\psi(l)$	current value the cross slope of the track at a distance l,	–
H	height from the level of the rails heads to a level of the any point on the vertical axis of the rail car cross section	mm
H_{Des}	height from the level of the rail heads to a level of the design point on the vertical axis of the rail car cross section	mm
H_{VAP}	height from the level of the rail heads to a design level of the vestibular apparatus of passenger of the rail car	mm
H_{MC}	height from the level of the rail heads to the level of the rail car mass center	mm
H_{CD}	height from the level of the rail heads to the level of the rail car coupling device	mm
H_{AB}	height from the level of the rail heads to the level of the axe box of the rail car bogie	mm
$j_H(l)$	current value of rate of change of the non-compensated lateral acceleration da_H/dt, acting at a height H at a distance l	m/s^3
J_{max}	maximum rate of change of non-compensated lateral acceleration, acting at height H_{VAP}	m/s^3
f(p)	function of the identical dependence of curvature k(l) and cant d(l) or cross slope $\psi(l)$ of transition on a proportion p=l/L, normalized in the range from 0 to 1	–
$f_k(p)$	function of the individual dependence of axle curvature of transition k(l) on a proportion p=l/L, normalized in the range from 0 to 1	–
$f_\psi(p)$	function of the individual dependence of track cross slope of transition $\psi(l)$ on a proportion p=l/L, normalized in the range from 0 to 1	–
$U\varepsilon_k(p)$	U-dependent corrections to the value of the function $f_k(p)$	–
$Z\varepsilon_\psi(p)$	Z-dependent corrections to the value of the function $f_\psi(p)$	–
τ_{max}	maximum value twist of the rail track over the transition	mm/m
W	dispersion of the oscillation amplitudes of the function dj_H/dt in the central part of the transition	m/s^4

Table 2. *Differences between the curvature of the track axis and that of the trajectory of the mass center "rigid" rail car from alignment styles of the transition.*

Style of the track alignment	Curvature k(l) of the axle of transition at level H = 0	Curvature $k_H(l)$ of path the mass center of railway car at level H ≥ 0
Conventional approach	$k(l) = K_B + f\left(\frac{l}{L}\right)\Delta K$	$k_H(l) \approx k(l) + \frac{d^2f}{dl^2}H\Delta\Psi$
For the trajectory of mass centre of the 'rigid' railway car moving at "out-swinging" transition curve	$k(l) \approx K_B + f\left(\frac{l}{L}\right)\Delta K - \frac{d^2f}{dl^2}H\Delta\Psi$	$k_H(l) = K_b + f\left(\frac{l}{L}\right)\Delta K$
For trajectory optimizing design point of 'rigid' railway car moving at harmonized transition	$k(l) = K_B + \left(f_k\left(\frac{l}{L}\right) + U\varepsilon_k\left(\frac{l}{L}\right)\right)\Delta K$	$k_H(l) \approx k(l) + \left(\frac{d^2f_\psi}{dl^2} + Z\frac{d^2\varepsilon_\psi}{dl^2}\right)H\Delta\Psi$

Over the last quarter century, many other functions f (p) have been proposed by different authors. Generally, these authors declare the expediency of their application for both conventional and nonconventional style of track alignment along the trajectory of the rail car mass center [13-15]. However, not all of the proposed functions can provide the generally recognized necessity of observing at least the G^2-th order of continuity of the curvature function of the "out-swinging" transition curve at its interface points with the curvature of adjacent track elements [12]. This conclusion follows from the analysis of the simplified formula of the curvature of "out-swinging" transition curve given in Table 2. It is clear that in order to satisfy the condition $d^2k/dl^2 = 0$ at both ends of the "out-swinging" transition curve, it is necessary use at least the G^4-th continuous function f(p). However, this order of continuity is redundant for the cross-slope function of the rail track and increases its twisting.

However, the G^2-th order of the continuity of the curvature of the "out-swinging" transition curve had a higher priority. Given this requirement, the G^4-th continuous function HHMP9 [12] and also the G^4-th continuous function Order (3,7) [13] were selected from all known alternatives for further analysis. The diagram of the HHMP9 function has the classic half-sine shape. This function is defined by a standard polynomial with the minimum 5-th degree of the first term and with the maximum 9-th degree of the last fifth term. The diagram of the function Order (3,7) has the form of a clothoid curvature with smoothed ends. It can also be called "half-sine with a linearized of the center". For analytical definition this function required a non-standard polynomial with the minimum 5-th degree of the first term and with the maximum 15-th degree of the last eleventh term. The shapes of the diagrams of these functions reflect the features of current trends in the search for transition curves with ideal performance properties for the track rounding of both conventional and high-speed railways [14, 15].

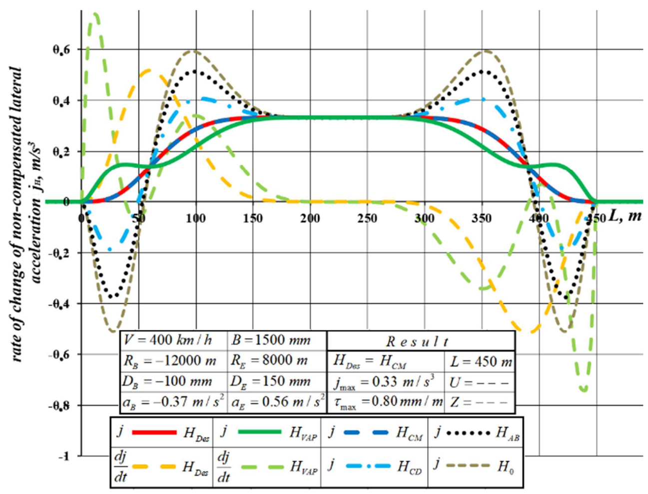

Figure 7. Integrative indicators of the quality of the reverse transition with a "out-swinging" transition curve of the HHMP9 type (Example N 4).

For a correct comparison, their integrative quality indicators were calculated using the same multifactor deterministic kinematic model of the curvilinear movement of a "rigid" rail car with variable curvature of the track axis and its cross slope [6], as in the above examples. The same predetermined values of the parameters V, R_B, D_B, R_E, D_E, H_{Des}, B, L, H_{VAP}, H_{CM}, H_{CD}, H_{AB}, J_{lim} and τ_{lim} were taken into account, as in the above Example N 1. The only difference was in the principle of assigning $H_{Des} = H_{CM}$ with the same reverse transition length L, which was established in the process of matching its shape with $H_{Des} = H_{VAP}$ in the above Example N 1. The results of these calculations are shown in Figures 7 and 8.

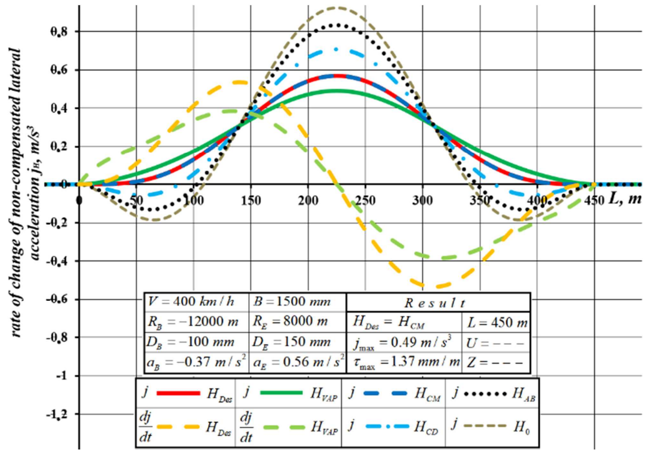

Figure 8. Integrative indicators of the quality of the reverse transition with a "out-swinging" transition curve of the Order (3.7) type (Example N 5).

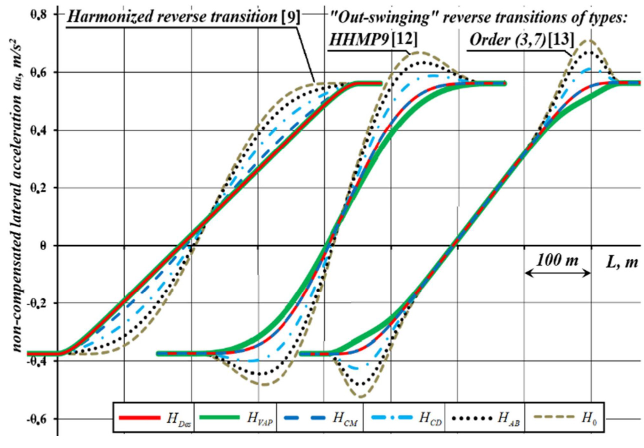

Figure 9. Diagrams of non-compensated lateral acceleration $a_H(l)$ acting at functionally significant levels of the considered alternatives (see Examples N 1, 4 and 5).

The results of calculations of the geometric and functional properties of alternative designs of reverse transitions were taken into account when ranking their priority. An additional argument to the conclusions arising from their analysis can serve as an illustration of their inherent diagrams of non-compensated lateral acceleration $a_H(l)$ (Figure 9). They correspond to the same conditions for their design and operation as in Examples N 1, 4 and 5.

5. Ranking Alternatives

The results of our calculations [7, 8] make it possible to rank the options of reverse transitions according to the properties important for the manufacturability of their construction, as well as for their subsequent operation and maintenance. They were obtained with the same length L, radius of curvature R, calculated cant D and movement speed V along the transitions of the analyzed alternatives. When evaluating their rank according to the length criterion L, the need of increasing the length of the constructions of reverse transitions with a “out-swinging” transition curve of the types of HHMP9 and Order (3, 7) was taken into account to achieve the results which are comparable with the results of harmonized transitions. Therefore, when evaluating their rank by the criterion of length L, which is necessary and sufficient to achieve the best results, the obvious fact was taken into account that alternative design of reverse transition with "out-swinging" transition curve of the types HHMP9 or Order (3, 7) need a longer length than the length L of its harmonized form.

The rank according to the criterion of passenger comfort was set taking into account the shapes of the diagrams $j_H(l)$ and their corresponding values of J_{max}. It was also believed that the smoothness and monotonicity of the change in non-compensated lateral acceleration acting at each of the H_{CM}, H_{CD}, H_{AB} levels are functionally significant for the quality of the force interaction of the Track + Rail Car system elements. Therefore, the rank for this aspect of the quality of its functioning was established taking into account the deviations of the graphs of the functions $j_H(l)$ from the ideal trapezoidal and/or acceptable bell-shaped shape and according to the values of τ_{max}.

Table 3. *Ratings of the properties of the analyzed reverse transitions alternatives.*

Geometric and functional properties	**Rating**		
	The harmonized reverse transitions	**The “out-swinging” reverse transition types:**	
		HHMP9	**Order (3, 7)**
Length L	1	2	3
Comfort J_{max}	1	3	2
Shape of the diagram $j_H(l)$ at the HMC level	1	2	3
Shape of the diagram $j_H(l)$ at the HCD level	1	2	3
Maximum twist of the track τ_{max}	2	3	1
Construction and maintenance	1	2	3
Average rating	1.17	2.33	2.5

The manufacturability of the device and the simplicity of maintaining the geometry of the reverse transitions in the design state were evaluated by the degree of their similarity to the natural shape of elastic rails, bent under the action of a smoothly and monotonically changing bending moment.

The evaluation of each of the alternatives by this or that property was set equal to 1 if this property most closely corresponded to the criterion adopted for it. As this correspondence decreased and the properties of transition deteriorated, the rank value increased to 2 or 3 (Table 3). The results of this assessment point to clear advantages of harmonized transition shapes over transitions with “out-swinging” transition curves like HHMP9 and Order (3, 7). These advantages are due to a number of differences in the geometry of these transitions and in the methods of harmonizing its properties with the design values of the design and operational parameters of the Track + Rail Car system.

The first difference consists in the use of non-identical functions of the curvature of the axis f_k and the track cross slope f_ψ of the transition of with different orders of their geometric smoothness G^4 and G^3, respectively. The coefficients of the main polynomial equations of these functions are substantiated from the condition of the best achievement of the optimization goal with a strictly monotonous change in the current values of the curvature of the axis k(l) and the track cross slope ψ(l) of the transition section of the rounding.

The second difference consists in the possibility of adapting the geometric properties of the main functions of the curvature of the axis f_k and the track cross slope f_ψ of the transition to the conditions of quasi-equilibrium movement of the car at $|a| \leq 0.3$ m/s^2. The need for such adaptation is objectively determined by the kinematics of the rail car design point, which is elevated above the level of the track. When designing a transition, such adaptation is implemented by summing the main values of the functions of the curvature of the axis f_k and the track cross slope f_ψ of the transition with the corrective values of the functions U×εk and Z×εψ (see Figure 3). The required values of the parameters U and Z are set together with the value of the transition length L in the process of their harmonization with the specified properties of the Track + Rail Car system.

The third difference consists in the presence of a formalized function of the purpose of harmonizing the properties of the Track + Rail Car system W and a numerical method for achieving it with W => MIN. At the calculated crew level $H_{Des} > 0$, this provides a quasi-linear shape of the diagram of uncompensated lateral accelerations $a_H(l)$ and the corresponding trapezoidal shape of the lateral jerk diagram $j_H(l)$. At the same time, the requirements of ergonomic and technical aspects of the quality of the functioning of the Track + Rail Car system at its other functionally significant levels $H \leq H_{Des}$ are also observed. This is confirmed by the quasi-linear shapes of the diagrams $a_H(l)$ and the bell-shaped diagrams of their rate of change $j_H(l)$.

6. New Principle for a Progressive Track Alignment Design

The above differences formed the basis of the provisions of the new principle of progressive alignment of normal, oval and reverse transitions. The existing principle reflects the experience of operating clothoid transitions with a linear cant of transition. It consists in observing the criterion $L \geq L_{min}$, in which L_{min} is set according to the limit value of one of the conditional indicators of the track quality ΔD/L, v×ΔD/L or v×ΔK/L. Generally, the most critical limit is set in terms of v×ΔD/L. The influence of the values of this indicator on the quality of the functioning of the Track + Rail Car system is not theoretically substantiated. Therefore, the recommended design values of this indicator take into account only the generalized experience of operating rail track with linear cant transitions and clothoids. They have a significant spread and can vary from 1/10 to 1/4 km/h.

A more adequate model [6] shows that the integrative indicators of the quality of the curvilinear movement of a rail car with a non-linearly changing trajectory curvature and lateral body roll do not have an explicit and direct relationship with the values of the differential indicator dd/dl. Its result depends on the combination of the main and derived values of the curvature functions of the track axis and its cross slope, as well as on the values of other functionally significant indicators of the Track + Rail Car system. Therefore, the identification of the limiting values of the indicators v × ΔD/L and v × dd/dl practiced in the standard [5] is incorrect. Their identical application in the general method for determining the minimum lengths L_{min} of transitions with half-sine and “out-swinging” transition curves is also incorrect.

In contrast to this, the provisions of the new principle of progressive alignment of the transition provide for the establishment of not a minimum, but an objectively justified length L, which is necessary and sufficient to achieve the goal W => MIN. At the same time, the values of the parameters U and Z are set, that increase the efficiency of this process. Its formalized goal W is represented by a functional that analytically describes the deterministic relationship of all functionally significant properties of the Track + Rail Car system: $W = F(V, K_B, K_E, D_B, D_E, B, H_{Des}, L, U, Z)$. These provisions are relevant for the harmonization of the forms of all types of normal, oval and reverse transitions.

When this task is implemented in software as a stand-alone function or built into the application system, the desired values of the parameters L, U and Z can be updated in real time after editing any of the above parameters. Diagrams of the functions $a_H(l)$ and $j_H(l)$ can also be quickly calculated and displayed. Along with visual control of their similarity to quasi-linear and/or trapezoidal shapes, the quantitative assessment of their quality by the value of the index W, as well as by compliance with the criteria $J_{max} \leq J_{lim}$ and $\tau_{max} \leq \tau_{lim}$ is more important. To assess compliance with other restrictions, these data can be supplemented by the values of displacement of the reverse circular parts of the rounding and the angles of the parts of the axis of the reverse transition with constant signs of curvature, as well as tangents and bisectors. It is also desirable to draw a plan of the reverse transition shown in the coordinate system of the project with obstacles important for its implementation.

Such a minimum of capabilities of this function will free the developers of the information model of the transport infrastructure object from routine calculations and create favorable conditions for more productive work to ensure high efficiency of its functioning. To a large extent, the success of this creative work depends on the conformity of the elemental base of the geometric design of transport infrastructure facilities with their functional features, as well as on taking these features into account in the criteria and methods for optimizing their designs.

7. Conclusion

The stated principles of optimizing reverse and other types of transitions are consistent with progressive recommendations for taking into account the kinematics of design points at functionally significant levels of the rail car [5]. They differ significantly from the principles for solving this problem using traditional and “out-swinging” transition curves. This is due to a systematic approach to harmonizing their properties in order to create prerequisites for a uniform change in accelerations and forces of interaction between the elements of the Track + Rail Car system. In contrast to the “out-swinging” transition curves, these goals are achieved with the natural bending of elastic rails with a smoothly and monotonously changing curvature. This simplifies their construction and maintenance geometry of the harmonized shape of the transitions in the proper state.

Joint harmonization of the length L of the transition and additional parameters of its shape U and Z with the design values V, R and D provides the best values of the integrative indicator of the dynamics of changes in accelerations and forces of interaction between the elements of the Track + Rail Car system. This indicator characterizes the maximum rate of their change J_{max}, which operates at the design level of the crew $H_{Des} \geq H_{VAP}$ for a given track width B. Its compliance with the requirements of ergonomic and technical aspects of the quality of operation of the system Track + Rail Car is assessed by the criterion $J_{max} \leq J_{lim}$. The safe value of torsion twisting of the track is estimated according to the traditional criterion $\tau_{max} \leq \tau_{lim}$.

The criteria, method and results of ensuring the quality of the functioning of the Track+Rail Car system by harmonizing all its significant properties is not consistent with the traditional practice, which is focused on observing only the allowable rate of change of cant dd/dt. Therefore, the lengths of transitions with non-constant gradients of the curvature and cross slope functions, that are set by the criterion dd/dt, may differ significantly from the necessary and sufficient lengths L of transitions, determined in the process of harmonizing their properties.

To improve the efficiency and quality of functioning of transitions with a progressive alignment design, this contradiction needs to be resolved, taking into account the results of their trial operation. Positive results of practical testing will eliminate the existing uncertainty in the norms and rules for designing railroad rounding with non-linear curvature functions k(l) and track cross slopes ψ(l) of their transition sections. To do this, it will be sufficient to introduce only the requirements for compliance with the trapezoidal shape of the diagram of the calculated slew rate of changes lateral accelerations, predicted at the H_{Des} level with $H_{Des} \geq H_{VAP}$ of the “rigid” rail car. Compliance with this requirement can be objectively assessed by the criterion $W \leq W_{lim}$. At the same time, the traditional requirements for ensuring passenger comfort, evaluated by the criterion $J_{max} \leq J_{lim}$ and safe torsion bending of the track, evaluated by the criterion $\tau_{max} \leq \tau_{lim}$, must also be observed.

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